$$Q = a_0 C_c v_2$$

$$= a_0 \times C_d \frac{\sqrt{\cancel{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{\cancel{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}}$$

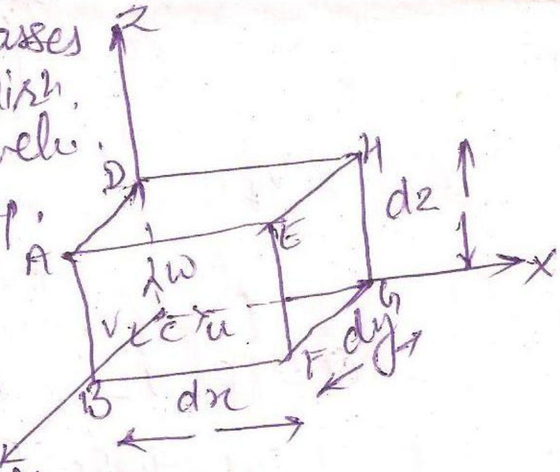
$$Q = \frac{a_0 \times C_d \times \sqrt{2gh}}{\sqrt{\frac{a_1^2 - a_0^2}{a_1^2}}}$$

$$\boxed{Q = \frac{C_d\, a_0\, a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}}$$

Unit 2

* Continuity Equation in Three dimensions :-

Consider the fluid element of masses dx, dy and dz in x, y & z dirn.
Let u, v, & w are the input velo. components in x, y & z dirn resp.



Mass of fluid entering at face $ABCD = \rho \times$ velo. in x dirn $\times$ Area of ABCD

$= \rho \times u \times dy\,dz$.

Mass of fluid leaving the face EFGH per second.

$= \rho u\,dy\,dz + \rho u \frac{\partial}{\partial x}(dy\,dz)\,dx$

$= \rho u\,dy\,dz + \frac{\partial}{\partial x} \rho u (dy\,dz)\,dx$

Gain of mass in x-dirn.

$=$ mass of ABCD $-$ mass leav. EFGH

$= \rho u\,dy\,dz - \rho u\,dy\,dz - \frac{\partial}{\partial x}(\rho u\,dy\,dz)\,dx$

$= -\frac{\partial}{\partial x}(\rho u)\,dx)\,dy\,dz$.

similarly net gain in y- dirn.

$= -\frac{\partial}{\partial y}(\rho v)\, dx\, dy\, dz$ [dydz = const.]

and in z dirn

$= -\frac{\partial}{\partial z}(\rho w)\, dx\, dy\, dz$.

Total Gain of mass

$$= -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx\, dy\, dz.$$

The mass is neither created nor destroyed. Thus net increase in mass per unit time must be equal to rate of increase of mass in fluid element. But mass in fluid element is $\rho\, dx\, dy\, dz$ and increase rate with time $= \frac{\partial}{\partial t}\rho\, dx.dy dz.$

or $\frac{\partial \rho}{\partial t}\, dx\, dy\, dz$.

Equating both masses

$$-\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] \cancel{dx\, dy\, dz} = \frac{\partial \rho}{\partial t} \cancel{dx\, dy\, dz}$$

$$\therefore \left(\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] + \frac{\partial \rho}{\partial t} = 0$$

for steady flow $\frac{\partial \rho}{\partial t} = 0$

$$\therefore \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

for incompressible flow ρ being const.

$$\therefore \boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

The above is the continuity equation in cartesian co-ordinates. For two dimn flow

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$