Homogeneous Functions

An expression in which every term is of the same degree is called homogeneous fun.

$$a_0 x^n + a_1 x^{n-1} y + a_2 x^{n-2} y^2 + \cdots \cdots + a_n y^n$$

is homogeneous fun. of x and y of degree n.
this can also be written as

$$x^n \left\{ a_0 + a_1\left(\frac{y}{x}\right) + a_2\left(\frac{y}{x}\right)^2 + \cdots \cdots + a_n\left(\frac{y}{x}\right)^n \right\}$$

i.e $x^n f(y/x)$ where $f(y/x)$ is function of $\left(\frac{y}{x}\right)$.

* To test a fun. $f(x,y)$ is homogeneous or not, we put tx for x and ty for y in it.
then fun becomes $f(tx, ty) = t^n f(x,y)$
here n shows the degree of homogeneous fun.

Euler's Theorem on Homogeneous fun.

<u>Statement</u>: If u is a homogeneous fun. of x and y of degree n, then $\boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu.}$

<u>Proof</u>: Since u is homogeneous fun. of x and y of degree n.

So, $u = x^n f(y/x)$ ——— (1)

$$\frac{\partial u}{\partial x} = x^n f'\left(\frac{y}{x}\right)\left(-\frac{y}{x^2}\right) + n x^{n-1} f(y/x).$$

$$\therefore x\frac{\partial u}{\partial x} = n x^n f\left(\frac{y}{x}\right) - x^{n-1} y \cdot f'(y/x) \quad \text{——— (2)}$$

Now $\frac{\partial u}{\partial y} = x^{n-1} f'(y/x)$

$$\therefore y\frac{\partial u}{\partial y} = x^{n-1} y\, f'(y/x) \quad \text{——— (3)}$$

Adding (2) and (3) we have,

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n x^n f(y/x)$$

$$\boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu}$$