

Problems on Jacobian & Taylor's, Maclaurin

1. If $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$, $z = r \cos \theta$, show that $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin \theta$ (MBU-2003)
2. If $u = \frac{y^2}{2x}$, $v = \frac{x^2 + y^2}{2x}$ find $\frac{\partial(u, v)}{\partial(x, y)} = -\frac{y}{2x}$ (MBU 2000, 2004)
3. If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1}x + \tan^{-1}y$ find $\frac{\partial(u, v)}{\partial(x, y)} = 0$
4. Expand: $f(x, y) = x^2y + 3y - 2$ in powers of $(x-1)$ and $(y+2)$ by Taylor's theorem. (MBU-2001)
5. Expand $e^x \cos y$ near the point $(1, \frac{\pi}{4})$ by Taylor's theorem.
6. Expand $f(x, y) = \tan^{-1}(\frac{y}{x})$ near the point $(1, 1)$ up to third degree terms compute $f(1.1, 0.9)$. Ans. : 6857