Problems on Euler's theorem

1. If $u = \log \frac{x^4+y^4}{x+y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$ (MDU 2004)

2. If $u = \tan^{-1} \frac{x^3+y^3}{x+y}$, Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. (MDU 2002)

3. If $u = \sin^{-1}\left(\frac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = -3\tan u$

4. If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$
$= -\frac{\sin u \cos 2u}{4\cos^3 u}$, (MDU 2000, 2004)

5. If $u = \operatorname{cosec}^{-1}\left(\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}\right)$, Prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$
$= \frac{\tan u}{144}(13+\tan^2 u)$.

(2001, 2003)

6. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, Prove $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin 4u - \sin 2u$

(MDU. 2001, 2002)

7. If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$.

(MDU - 2002, 2003)

8. Show that $xu_x + yu_y + zu_z = 2\tan u$. where

$$u = \sin^{-1}\left(\frac{x^3+y^3+z^3}{ax+by+cz}\right).$$