Divide eqⁿ (9) by (10)

$$\frac{x_0 \sin\phi}{x_0 \cos\phi} = \frac{-2\xi_1 r\, k x_i}{(1-r^2)^2 - (2\xi_1 r)^2} \times \frac{(1-r^2)^2 - (2\xi_1 r)^2}{k x_i (1-r^2)}$$

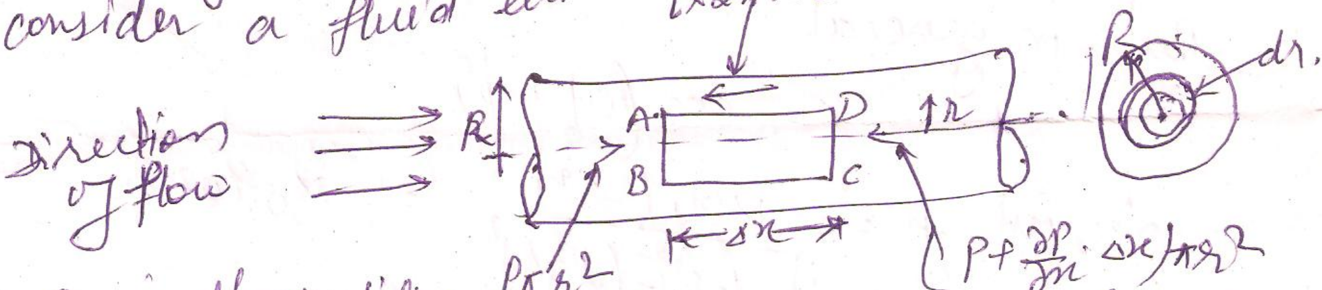
$$\boxed{\tan\phi = -\frac{2\xi_1 r}{(1-r^2)}}$$

F.M

Unit-5 / viscous Flow

* Relation Between shear stress, pressure gradient and velocity distribution :-

consider a horizontal pipe of radius R. The viscous fluid is flowing from left to right as shown in fig. consider a fluid element



of radius r is ~~flowt~~ sliding in cylindrical fluid element of radius (r+dr). let p is the pressure at face AB & $(p+\frac{\partial p}{\partial x}\cdot\Delta x)$ is pressure at CD. the pressure forces acting on fluid element ABCD

1. Pressure force on AB $= p\pi r^2$
2. " " " face CD $= (p+\frac{\partial p}{\partial x}\cdot\Delta x)\pi r^2$
3. shear force $\tau \times 2\pi r \Delta x$ on surface on the

As there is no acceleration therefore the summation of pressure forces will be zero

$$\therefore\ p\pi r^2 - (p+\frac{\partial p}{\partial x}\cdot\Delta x)\pi r^2 - (\tau \times 2\pi r \Delta x) = 0$$

$$-\frac{\partial p}{\partial x}\cdot\Delta x\,\pi r^2 - \tau\, 2\pi r \Delta x = 0$$

$$-\frac{\partial p}{\partial x}\cdot r - \tau 2 = 0 \Rightarrow \boxed{\tau = -\frac{\partial p}{\partial x}\cdot\frac{r}{2}}$$

Now velocity distribution

we know $\tau = \mu \frac{du}{dy}$, but here y is measured from pipe wall

$\therefore y = R - r$ or $dy = -dr$

$\tau = -\mu \frac{du}{dr}$ substituting — ①

$$-\frac{\partial p}{\partial x} \cdot \frac{r}{2} = -\mu \frac{du}{dr}$$

$$\Rightarrow \frac{\partial p}{\partial x} \frac{r}{2} = \mu \cdot \frac{du}{dr} \quad \text{or} \quad \mu \frac{du}{dr} = \frac{\partial p}{\partial x} \cdot \frac{r}{2}$$

As $\frac{\partial p}{\partial x}$ is constt. Integrating w.r.t r

$$\therefore \int du = \int \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right) \cdot r \cdot dr$$

$$\Rightarrow u = \frac{1}{2\mu} \frac{\partial p}{\partial x} \cdot \frac{r^2}{2} + C$$

$$\therefore u = \frac{1}{4\mu} \frac{\partial p}{\partial x} r^2 + C$$

for finding C, put $r = R$ & $u = 0$

$$0 = \frac{1}{4\mu} \frac{\partial p}{\partial x} R^2 + C$$

$$\therefore u = \frac{1}{4\mu} \frac{\partial p}{\partial x} r^2 - \frac{1}{4\mu} \frac{\partial p}{\partial x} R^2$$

$$= -\frac{1}{4\mu} \frac{\partial p}{\partial x} \left[R^2 - r^2\right]$$

Ratio of max. to average velocity :—

$$U_{max} = -\frac{1}{4\mu} \frac{\partial p}{\partial x} R^2$$

Average velocity $= Q/\text{Area}$

$dQ =$ velocity at radius $r \times$ element ring area

$$= u \times 2\pi r dr$$

$$= -\frac{1}{4\mu} \frac{\partial p}{\partial x} (R^2 - r^2)\, 2\pi r\, dr$$

$$= -\frac{\pi}{2\mu} \frac{\partial p}{\partial x} (R^2 r - r^3)\, dr$$

$$Q = \int_0^R \frac{\pi}{2\mu}\left(-\frac{\partial p}{\partial x}\right) \left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_0^R$$

$$\Rightarrow Q = \frac{\pi}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$Q = \frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^4$$

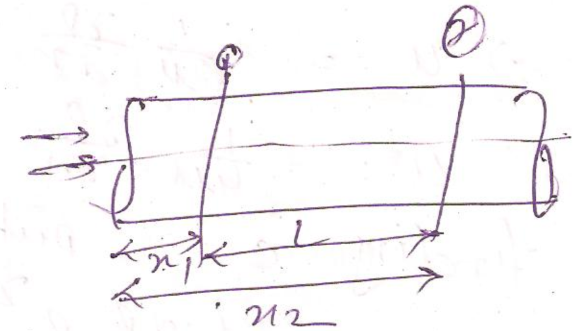
$$\bar{u} = \frac{\frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^4}{\pi R^2} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^2$$

$$\therefore \frac{u_{max}}{\bar{u}} = \frac{-\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) R^2}{-\frac{1}{8\mu}\frac{\partial p}{\partial x} R^2} = \underline{2.0}$$

* Drop of pressure for a given length (L) of a pipe:-

we know

$$\bar{u} = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^2$$



$$-\frac{\partial p}{\partial x} = \frac{8\mu\bar{u}}{R^2}$$

Integrat w.r.t x -

$$-(P_1 - P_2) = \frac{8\mu\bar{u}}{R^2}\cdot[x_1 - x_2]$$

$$\Rightarrow (P_1 - P_2) = \frac{8\mu\bar{u}(x_2 - x_1)}{R^2} \qquad [\because x_2 - x_1 = L]$$

$$(P_1 - P_2) = \frac{8\mu\bar{u}}{R^2}\cdot L = \frac{8\mu\bar{u}L}{(D/2)^2}$$

$$= \frac{32\mu\bar{u}\cdot L}{D^2}$$

But loss of pressure head $\frac{P_1 - P_2}{\rho g} = H_f$

$$\therefore \boxed{\frac{P_1 - P_2}{\rho g} = \frac{32\mu\bar{u}L}{\rho g D^2}}$$

The above is called Halgen Poiseuille formula