* Relation b/w velocity distribution & press. gradient of two parallel plates :-

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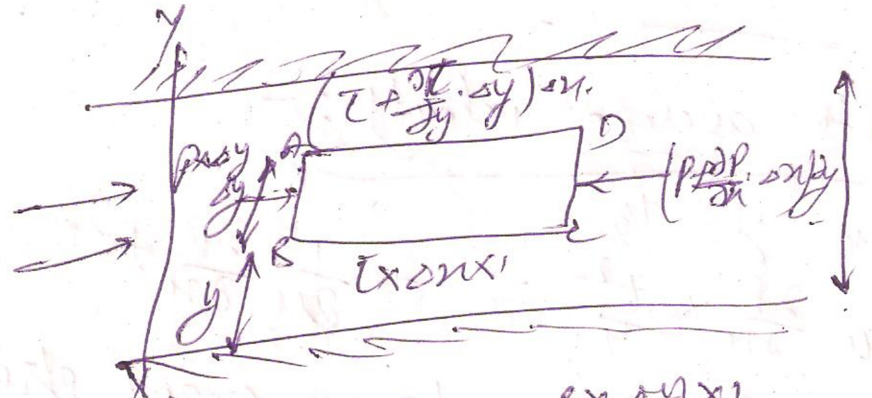

Pressure force on AB = $p \times \delta y \times 1$.

CD = $\left(p + \frac{\partial p}{\partial x} \cdot \delta x\right) \delta y \times 1$.

BC = $\tau \times \delta x \times 1$.

AD = $\left(\tau + \frac{\partial \tau}{\partial y} \cdot \delta y\right) \delta x$.

for eqm

$$p \times \delta y \times - \left(p + \frac{\partial p}{\partial x} \cdot \delta x\right) \delta y - \tau \delta x + \left(\tau + \frac{\partial \tau}{\partial y} \cdot \delta y\right) \delta x = 0$$

$$\Rightarrow - \frac{\partial p}{\partial x} \cdot \delta x \, \delta y - \frac{\partial \tau}{\partial y} \delta y \, \delta x = 0$$

$$= \frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}.$$

$$\frac{\partial p}{\partial x} = \frac{\partial \left(\mu \frac{du}{dy}\right)}{\partial y} = \mu \frac{\partial^2 u}{\partial^2 y}.$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu} \cdot \frac{\partial p}{\partial x}$$

$$\frac{\partial u}{\partial y} = \frac{1}{\mu} \frac{\partial p}{\partial x} y + C_1$$

$$u = \frac{1}{\mu} \frac{\partial p}{\partial x} \frac{y^2}{2} + C_1 y + C_2$$

at $y = 0$, $u = 0$, at $y = t$, $u = 0$, $C_2 = 0$

$$\Rightarrow 0 = \frac{1}{\mu} \frac{\partial p}{\partial x} + C_1 \times t \qquad C_1 = -\frac{1}{\mu} \frac{\partial p}{\partial x} \cdot t.$$

$\Rightarrow$

$$u = \frac{1}{\mu}\frac{\partial p}{\partial x}\cdot\frac{y^2}{2} - \frac{1}{\mu}\frac{\partial p}{\partial x}\cdot\frac{t}{2}\times y$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left(ty - y^2\right)$$

Ratio of max. to average velocity :-

$u_{max.}$ = when $y = t/2$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\times\frac{t^2}{4} = -\frac{1}{8\mu}\frac{\partial p}{\partial x}t^2$$

~~Avg velocity~~ dQ = velocity at dist. y × area of strip.

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}(ty - y^2)\times dy\times 1$$

$$Q = \int_0^t -\frac{1}{2\mu}\frac{\partial p}{\partial x}(ty - y^2)\,dy$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{t^3}{6}\right] = -\frac{1}{12\mu}\frac{\partial p}{\partial x}t^3$$

$$\bar{u} = \frac{Q}{A} = \frac{-\frac{1}{12\mu}\frac{\partial p}{\partial x}t^3}{t\times 1} = -\frac{1}{12\mu}\frac{\partial p}{\partial x}t^2$$

$$\frac{u_{max.}}{\bar{u}} = \frac{-\frac{1}{8\mu}\frac{\partial p}{\partial x}t^2}{-\frac{1}{12\mu}\frac{\partial p}{\partial x}t^2} = \frac{3}{2}$$

Drop of pressure

$$\bar{u} = -\frac{1}{12\mu}\frac{\partial p}{\partial x}t^2$$

$$\Rightarrow \frac{\partial p}{\partial x} = -\frac{12\mu\bar{u}}{t^2}$$

$$\int_2^1 \partial p = \int_2^1 \frac{-12\mu\bar{u}}{t^2}\,\partial x = \frac{-12\mu\bar{u}(x_1 - x_2)}{t^2}$$

$$(P_1 - P_2) = \frac{12\mu\bar{u}L}{t^2} \Rightarrow \frac{P_1 - P_2}{\rho g} = \frac{12\mu\bar{u}L}{\rho g t^2}$$

$$\tau = \mu\frac{du}{dy} = -\frac{1}{2}\frac{\partial p}{\partial x}[t - 2y]$$

$$\tau_{max.} = -\frac{1}{2}\frac{\partial p}{\partial x}\cdot t$$

at $y = 0$