B.Tech.

Third Semester Examination,

Discrete Structure (CSE-203-F)

Note : Attempt any ***FIVE*** questions. All questions carry equal marks.

Q. 1. (a) Define the following terms and give examples of each :

Countable Set, Symmetric difference, POSET, Bounded Lattice, Supremum.

Ans. Countable Set : Set is a collection of well defined objects and countable set includes the set whose member elements can be counted.

For Example : The collection of vowels in English alphabets.

Symmetric Difference : Symmetric difference of A and B is defined as the set of all elements that belong to A or B, bit not to both A and B, and we denote it by $A \oplus B$.

Example: $A \oplus B = \{(x_1 x \in A \text{ and } x \notin B) \text{ or } (x \in B \text{ and } x \notin A)\}$

POSET : A relation R in a set P is called a partial order set if it satisfies reflexive, Antisymmetric and transitive properties.

Bounded Lattice : A poset $(S, \leq)$ is called a bounded lattice if every two-element subset of S have both a least upper bound and a greatest lower bound.

Supremum : The maximum value attained after satisfying all conditions of a relation being a POSET and a lattice is called supremum.

Q. 1. (b) Prove the following identities :

(i) $(A \cup B) \cup (A \cap B^C) = A$

(ii) $(A - C) \cap (C - B) = \phi$

(iii) $(A - B) \subseteq A$

(iv) $(A - B) \cup (A \cap B) = A$.

Ans. (i) $(A \cup B) \cup (A \cap B^C) = A$:

Let x be an arbitrary element of $(A \cup B)$ then,

$$(x \in A \text{ or } x \in B)$$

$$\Rightarrow \quad x \in A \text{ or } x \in (A \cup B)$$

$$\Rightarrow \quad x \in B \text{ or } x \in (A \cup B)$$

$$\Rightarrow \quad (A \cup B) \cup (A \cap B^C) = A$$ **Hence proved**

(ii) $(A-C)\cap(C-B)=\phi$:

If possible set $(A-C)\cap(C-B)\neq\phi$. Then there exists at least one element x, say in $(A-C)\cap(C-B)$.

$$x\in(A-C)\cap(C-B)\Rightarrow x\in(A-C) \text{ and } x\in(C-B)$$

$$\Rightarrow \quad (A-C)\cap(C-B)=\phi.$$

(iii) $(A-B)\subseteq A$:

$$\{(A\cap B)\cap(A)\}\subseteq A$$

$$\Rightarrow \quad (A-B)\subseteq A$$

(iv) $(A-B)\cup(A\cap B)=A$:

$A-B\in$ to all elements which are A but not in B. $A\cap B\in$ to all elements which are common in A and B.

$$\therefore \quad (A-B)\cup(A\cap)B=A.$$

Q. 1. (c) If A is a set of people and R is a relation defined on A as :

$$R=\{(x,y):(x,y)\in A\times A \text{ and } x \text{ is a sister of } Y\}$$

Verify if R is symmetric?

Ans. Let R be a relation on a set P, let R be a subset of $P\times P$, then R is said to be symmetric relation. If $(a,b)\in R\Rightarrow(b,a)\in R$. Then R is symmetric if we have b R a whenever aRb.

If A is a set of people and R is a relation and is means is sister of and aRb may be the sister of a. Here, aRb implies bRa.

$\therefore$ R is symmetric.

Q. 2. (a) Explain the following types of relations with suitable examples.

Reflexive Relation, Symmetric Relation, Transitive relations, Equivalence Relation, Circular Relation.

Ans. Reflexive Relation : Let R be a relation on a set P, let R be a subset of $P\times P$. Then R is called a Reflexive relation if $(a,a)\in R\ \forall a\in P$. Thus, R is reflexive if we have $aRa\ \forall a\in A$.

Symmetric Relation : Let R be a relation on a set P, let R be subset of $P\times P$. Then R is said to be symmetric relation if $(a,b)\in R\Rightarrow(b,a\in R)$.

Transitive Relation : Let R be a relation on a set P. Let R be a subset of $P\times P$. Then R is said to be transitive relation if $(a,b)\in R$ and $(b,c)\in R\Rightarrow(a,c)\in R$.

Equivalence Relation : Let R be a relation in a set P. Then R is an equivalence relation in P if and only if,

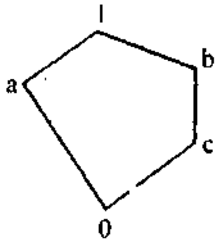
(i) R is reflexive

(ii) R is symmetric

(iii) R is transitive.

Circular Relation : When properties of first element coincide with that of last element to form a closed structure, it is called circular relation.

Q. 2. (b) Verify whether the lattice given by following Hasse diagram is distributive.



Ans. A lattice L is distributive for any elements a, b and c in L, if we have

(i) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

(ii) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

$\therefore$ It is not distributive.

Q. 2. (c) If R is a relation on the set of integers such that $(a, b) \in R$ if and only if $b = a^m$ for some positive integer m_0 show that R is partial ordering.

Ans. For a relation to be partially order it should be :

(i) Reflexive

(ii) Antisymmetric

(iii) Transitive

$\therefore$ R is a rela: $(a, b) \in R$ if and only if $b = a^m$ for some positive integer m_0.

$\therefore$ $$b = a^1$$
$$b = a^2$$

But $b \neq a^{-1}$ and $b \neq a^{-2}$

$\therefore$ It is not symmetric.

$\therefore$ It is not partially ordered.

Q. 3. (a) A group of five students say A, B, C, D and E have decided to talk with the Maths Department chairperson about their queries. The chairperson has said that she will speak with three of the students. In how many ways can these five students choose three of their group to talk with the chairperson.

Ans. Number of students = 5

Chair person chooses to talk to 3 at one times.

$\therefore$ They can choose 5 groups to talk with the chair person.

Q. 3. (b) If the number of diagonals in a polygon is 44, then find the number of sides in the polygon.

Ans. Number of diagonals in a polygon = 44

Number of sides $= {}^{a}p$

$= {}^{a}4^{b}4$

$= {}^{b}22_{44}$

$= 22.$

Q. 3. (c) How many distinct four digit integers can one make from digits 1, 3, 3, 7, 8, 8.

Ans. Number of distinct four digit integers are $= 4^6$.

Q. 4. (a) Define each of following and explain with example :

Field, Coset, Monoid, Iosmorphism, Ideal.

Ans. Field : A non empty set G equipped with one or more binary operations is called field.

Coset : Suppose G is a group and H is any sub-group of G. Let a be any element of G. Then the set $Ha = \{ha : h(-x)\}$ is called right coset of H in G generated by a. Similarly the set $aH = \{ah : h \in H\}$ i.e., called a left coset of H in Q generated by a.

Monoid : An algebraic structure (G, 0) is called a monoid if the following properties are satisfied :

(i) The binary operation is closed operation.

(ii) The binary operation is association.

(iii) There exists an identity element.

Isomorphic : Suppose G and G' are two groups, the compensation in each being denoted multiplicatively. A mapping $f : G \to G'$ is said to be an isomorphic mapping of G into G'. If

(i) f is one-one.

(ii) $f(ab) = f(a)f(b)\ \forall a, b \in G$.

Ideal : If in a ring R there exists an element I such that $I.a = a = a.I \quad \forall a \in R$, then R is called ideal with unit element.

Q. 4. (b) State and prove Euler's formula for planar graphs.

Ans. Step 1 : Choose any vertex V_0 in the Euler graph G and set $W_0 = V_0$.

Step 2 : If the trail $W_i = V_0, e_1, v_1 .... e_i, v_i$ has been chosen, choose an edge $e_i + 1$ different from $e_1 ...., e_i$ such that,

(i) $e_i + 1$ is incident with V_i.

(ii) Unless there is no alternative $e_i + 1$ is not a bridge of the edge deleted subgraph $G - \{e_1 - e_i\}$.

Step 3 : Step. if W_i contains every edge of G, otherwise repeat step 2.

$$|v| - |E| + |R| = 2.$$

Q. 4. (c) State and prove Lagrange's theorem.

Ans. Let G be a group of finite order n and H be a sub-group of G.

Let $\quad O(H) = m$

$$G = Ha_1 \cup Ha_2 \cup Ha_3 \cup ..... \cup Ha_k$$

$\Rightarrow \quad O(G) = km \Rightarrow n = km$

$\Rightarrow \quad k = \frac{n}{m} \Rightarrow$ m is a dimension of n.

Q. 5. (a) Find the discrete numeric function corresponding to generating function :

$$G(x) = \frac{x^2 + 2x}{1 - x - x^2}.$$

Ans.

$$G(x) = \frac{x^2 + 2x}{1 - x - x^2}$$

$$= -\left[\frac{x(x+2)}{x^2 + x - 1}\right]$$

$$= -\left[\frac{x(x+2)}{x^2 + x - 1}\right]$$

$$= -\left[\frac{x}{(x^2 + x - 1)} \cdot \frac{x+2}{x^2 + x - 1}\right]$$

$\because$ It cannot be further decomposed, it does not have a numeric function.

Q. 5. (b) Prove that

$$(p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$$

Ans. $(p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$

$\Rightarrow$ $(p \vee q) \to r$

$\Rightarrow$ $p \vee q$ and $q \vee r$

$\Rightarrow$ $p \to r$ and $q \to r$

$\Rightarrow$ $(p \to r) \wedge (q \to r) = \text{R.H.S.}$

Hence Proved.

Q. 5. (c) Solve the recurrence relation,

$$a_n - n\,a_{n-1} = n! \text{ for } n \geq 1,\ a_0 = 2.$$

Ans. $a_n - na_{n-1} = n!$ for $n \geq 1$, $a_0 = 2$

$a_n - na_{n-1} = n!$

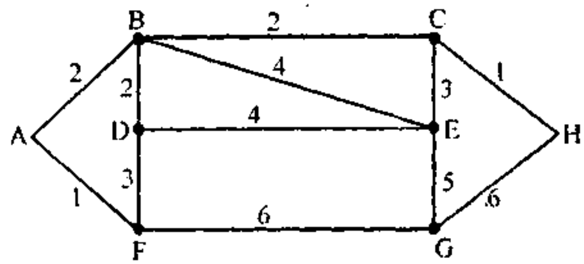
$a_1 - |a_0 = 1|!$

$\Rightarrow$ $a - 1 + 2a = i$

$= 1 = 1$

$\therefore$ It is a recurrence relation.

Q. 6. (a) Use Dijkstra's algorithm to find the shortest path between vertices A and H in the weighted graph given below :



Ans. Shortest path = Sum of edges should be least

= A to B

Then B to C

Them C to H

$= 2 + 2 + 1$

$= 5$

∴ Shortest path = ABCH.

Q. 6. (b) What is meant by minimum spanning trees. Explain Kruskal's method to find minimum spanning tree in a graph.

Ans. A spanning sub-graph of graph G that is a tree is called spanning tree of G.

Q. 6. (c) Explain the concept of cut-set, cut-vertex and bridge with examples.

Ans. Cut-Set : The elements obtained after applying an operational is called cut-set.

Cut-Vertex : The edges obtained by minimal spanning tree is called cut-vertex.

Bridge : The shortest path formed by operation is called bridge.

Q. 7. (a) Show that number of odd degree vertices in any undirected graph is even.

Ans. In an undirected graph let numbers of odd degree vertices = n

∴ Even vertices in an undirected graph

$= n - 1$

∴ Proved.

Q. 7. (b) Prove Demorgan's laws of boolean algebra.

Ans. If X and Y are any two sets, then

(i) $(X \cup Y)' = X' \cap Y'$

(ii) $(X \cap Y)' = X' \cup Y'$

Q. 7. (c) Is there any Hamiltonian path/circuit in $K_{4,4}$, $K_{4,5}$ and $K_{4,6}$, explain your answer.

Ans. Yes, there is Hamiltonian path in $K_{4,4}$, $K_{4,5}$ and $K_{4,6}$ because it consecutively forms, a path without any break.

Q. 8. (a) Find the sum of following series :

$0.3 + 0.33 + 0.333 + 0.3333 + \ldots\ldots$

Ans. $0.3 + 0.33 + 0.333 + 0.3333 + \ldots\ldots$

$$0.3 + (0.3)^n + (0.3)^{n+1} + \ldots\ldots$$

$\Rightarrow$ 0.3^{n^n} .

Q. 8. (b) A tree has 2n vertices of degree 1, 3n vertices of degree 2 and n vertices of degree 3. Determine the number of vertices in the tree and number of edges in the tree.

Ans. Total number of vertices $= 2n + 3n + n$

$= 6n$

Total number of edges $= n$.

Q. 8. (c) Classify the following proportions into tautology, absurdity or contingency by constructing truth tables :

(i) $p \to q \wedge p$

(ii) $(p \wedge q) \vee (q \wedge \sim p)$

(iii) $p \wedge \sim p$

(iv) $q \to (q \to p)$

Ans. (i) $p \to q \wedge p$: $p \to q \wedge p$

It is a tautology.

(ii) $(p \wedge q) \vee (q \wedge \sim p)$:

It is a tautology.

(iii) $p \wedge \sim p$:

It is a convergency.

(iv) $q \to (q \to p)$:

It is a contingency.