B.E.

Third Semester Examination, May-2007

DISCRETE STRUCTURE

Note : Attempt only five question.

Q. 1. (a) Determine if each function is ONE-to-ONE :

(i) To each person on the earth assign the number which correspond to his age.

(ii) To each country in the world assign altitude and longitude of its capital.

(iii) To each book written by only one author assign the author.

(iv) To each country in the world which has a prime minister assign its prime minister.

Ans.

(i) To each person on the earth assign the number which correspond to his age.

This is not one-to-one function.

(ii) To each country in the world assign latitude and longitude of its capital. This is one-to-one function.

(iii) To each bok written by only one author assign the author. This is one-to-one function.

(iv) To each country in the world which has a prime-minister assign its prime-minister. This is one-to-one function.

Q. 1. (b) Prove :

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Ans. (i) Taking L.H.S.,

$A \times (B \cap C)$

Let $(x, y) \varepsilon A \times (B \cap C) \Rightarrow x \varepsilon A$ and $y \varepsilon B \cap C$

$\Rightarrow$ $x \in A$ and $(y \varepsilon B \text{ and } y \varepsilon C)$

$\Rightarrow$ $x \varepsilon A$ and $y \varepsilon B$ and $x \in A$ and $y \in C$

$\Rightarrow$ $(x, y) \varepsilon A \times B$ and $(x, y) \varepsilon A \times C$

$\Rightarrow$ $(x, y) \varepsilon (A \times B) \cap (A \times C)$

$\therefore$ $A \times (B \cap C) \subset (A \times B) \times (A \times C)$ ...(i)

Conversely let $(x, y) \varepsilon (A \times B) \times (A \times C)$

$\Rightarrow \quad (x,y)\varepsilon A\times B$ and $(x,y)\varepsilon A\times C$

$\Rightarrow \quad (x\varepsilon A$ and $y\varepsilon B)$ and $(x\varepsilon A$ and $y\varepsilon C)$

$\Rightarrow \quad x\varepsilon A$ and $y\varepsilon B$

$\Rightarrow \quad x\varepsilon A$ and $y\varepsilon B\cap C$

$\Rightarrow \quad (x,y)\varepsilon A\times(B\cap C)$

$\Rightarrow \quad (A\times B)\times(A\times C)\subset A\times(B\cap C)$ ...(ii)

From (i) & (ii)

$$A\times(B\cap C)=(A\times B)\cap(A\times C)$$

Hence proved.

(ii) Let $(x,y)\varepsilon A\times(B\cap C)$

$\Rightarrow \quad x\in A$ and $y\varepsilon B\cup C$

$\Rightarrow \quad x\varepsilon A$ and $(y\varepsilon B$ or $y\varepsilon C)$

$\Rightarrow \quad (x\varepsilon A$ or $y\varepsilon B)$ or $(x\varepsilon A$ and $y\varepsilon C)$

$\Rightarrow \quad (x\varepsilon A$ and $y\varepsilon B)$ on $(x\varepsilon A$ and $y\varepsilon C)$

$\Rightarrow \quad (x,y)\varepsilon A\times B$ or $(x,y)\varepsilon(A\times C)$

$\Rightarrow \quad (x,y)\varepsilon(A\times B)\cup(A\times C)$

$\therefore \quad A\times(B\cup C)\subset(A\times B)\cup(A\times C)$ ..(i)

Conversely let $(x,y)\varepsilon(A\times B)\cup(A\times C)$

$\Rightarrow \quad (x,y)\varepsilon(A\times B)$ or $(x,y)\varepsilon(A\times C)$

$\Rightarrow \quad (x\varepsilon A$ and $y\varepsilon B)$ or $(x\varepsilon A$ and $y\varepsilon C)$

$\Rightarrow \quad x\varepsilon A$ and $y\varepsilon B$ or $y\varepsilon C$

$\Rightarrow \quad x\varepsilon A$ and $y\varepsilon(B\cup C)$

$\Rightarrow \quad (x,y)\varepsilon A\times(B\cup C)$

$\therefore \quad (A \times B) \cup (A \times C) \subset A \times (B \cup C)$ ...(ii)

From (i) & (ii)

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$

Hence proved.

Q. 1. (c) Consider the following assumptions :

S1 : All dictionaries are useful.

S2 : Mary owns only romance novels.

S3 : No romance novel is useful.

Determine the validity of each of the following conclusions :

(i) Romance novels are not dictionaries.

(ii) Mary does not own a dictionary.

(iii) All useful books are dictionaries.

Ans. (i) Valid

(ii) Not valid

(iii) Not valid

Q. 2. (a) Check for the tautology of the statements given below :

If I look into the sky and I am alert then either I will see the flying saucer or if 10. I am not alert then I will not see the flying saucer.

Ans. Let P be "I am alert" and q be "I will see the flying source". Then above argument can be written in symbolic form as follows :

$$p \to -q$$
$$q$$
$$-p$$

p	q	−p	−q	p →~ q
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

$p \to \sim q$ is true in line 2, 3 and 4. q is true in line 1 and 4 and ~ p is time in line 3 and 4. Hence, all three are time in line 4. So it is a valid statement.

Q. 2. (b) Express ↓ in terms of 1.

Ans.

NOR			OR
Alert	Seen	O/P	O/P
1	1	0	1
1	0	0	1
0	1	0	1
0	0	1	0

Q. 2. (c) Express l in terms of ↓.

Ans.

	OR		NOR
Alert	Seen	Output	O/p
1	1	1	0
1	0	0	0
0	1	0	0
0	0	0	1

Q. 2. (d) Express $\wedge$, V and .. in terms of NOR only.

Ans. According to the strut given AND into NOR.

A	B	C
0	*0*	*0*
0	1	0
1	0	0
1	1	1

According to principle of duality changing the I/P states we got

A	B	C
1	1	1
1	0	0
0	1	0
0	0	0

OR into NOR

A	B	C	$\overline{C}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	*0*

NOT into NOR

A	$\bar{A}$
0	1
1	0

Q. 3. In a class of 100 students, 40 were boys :

(a) In how many ways can a 10 person committee be formed?

Ans. Total students are 100 boys are 40.

Girls are $= 100 - 40 = 60$

Now, there are 100 students and we have to form committee of 10 students. So various combinations can be,

$$^{100}C_{10} \Rightarrow \frac{100!}{(100-10)!*10!} = \frac{100 \times 99 \times 98 \times *91 \times 90!}{90!*10!}$$

$$= 100 \times 99 \times 98*97*96*95*94*93*92*91/10!$$

$$\frac{100*99*98*97*96*95*94*93*92*91}{10*9*8*7*6*5*4*3*2*1}$$

$$11*98*97*95*47*31*92*13$$

Q. 3. (b) Repeat part (a) if there must be an equal no. of boys and girls in the committee.

Ans.

$$\frac{^{40}C_5 *^{60}C_5}{^{100}C_{10}} = \frac{\frac{40!}{40-5!*5!} * \frac{60!}{60-5!*5!}}{\frac{100!}{100-10!*10!}}$$

$$= \frac{\frac{40*}{36!*6!} \times \frac{60!}{55! \times 5!}}{\frac{100!}{90! \times 10!}}$$

$$\frac{\frac{40 \times 39 \times 39 \times 37 \times 36 \times 35}{35! \times 5!} \times \frac{60 \times 59 \times 58 \times 57 \times 56 \times 55}{55! \times 5!}}{100 \times 99 \times 98 \times 97 \times 96 \times 95 \times 94 \times 93 \times 92 \times 91 \times 90!}$$

$$= \frac{40 \times 39 \times 38 \times 37 \times 36 \times 35 \times 34 \times 33 \times 32 \times 31}{5 \times 4 \times 3 \times 2 \times 1 \times 5 \times 4 \times 3 \times 2 \times 1 \times 100 \times 99 \times}$$

$$98 \times 97 \times 96 \times 95 \times 94 \times 93 \times 92 \times 91$$

$$= \frac{32476491}{3686210 \times 49}$$

Q. 3. (c) Repeat part (a) if the committee must consist of either six boys and four girls or four boys and six girls.

Ans.

$$\frac{{}^{60}C_4 \times {}^{40}C_6}{{}^{100}C_{10}} + \frac{{}^{40}C_4 \times {}^{60}C_6}{{}^{100}C_{10}}$$

$$\frac{\frac{60!}{56! \times 4!} \times \frac{40!}{34! \times 6!}}{\frac{100!}{90! \times 10!}} + \frac{\frac{40!}{36! \times 4!} \times \frac{60!}{54! \times 6!}}{\frac{100!}{90! \times 10!}}$$

$$\frac{+\frac{60 \times 59 \times 58 \times 57 \times 56!}{56! \times 4 \times 3 \times 2 \times 1} \times \frac{40 \times 39 \times 38 \times 37 \times 36 \times 35 \times 34 \times 33 \times 32 \times 31}{34! \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}}{\frac{100 \times 99 \times 98 \times 97 \times 96 \times 95 \times 94 \times 93 \times 92 \times 91 \times 90}{90! \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}}$$

$$= \frac{\frac{60 \times 59 \times 58 \times 57 \times 40 \times 39 \times 38 \times 37 \times 36}{6 \times 5 \times 4 \times 3 \times 4 \times 3 \times 2} + \frac{60 \times 59 \times 58 \times 57 \times 56 \times 55 \times 54 \times 53 \times 52 \times 51}{6 \times 5 \times 4 \times 3 \times 2 \times 4 \times 3 \times 2}}{\frac{11 \times 98 \times 97 \times 96 \times 95 \times 94 \times 93 \times 92 \times 91}{8 \times 7 \times 6 \times 4 \times 3 \times 2}}$$

$$= \frac{59 \times 58 \times 57 \times 10 \times 39 \times 38 \times 37 \times 59 \times 58 \times 57 \times 14 \times 55 \times 6 \times 5 \times 39 \times 38 \times 37 \times 3}{11 \times 7 \times 97 \times 95 \times 94 \times 93 \times 96 \times 91} = \frac{59 \times 29 \times 570 \times 39 \times 38 \times 37 \times 59 \times 58 \times 57 \times 14 \times 6 \times 5 \times 3 \times 37}{7 \times 97 \times 47 \times 31 \times 23 \times 7}$$

Q. 4. (a) Let $4a_r + C_1 a_{r-1} + C_2 a_{r-2} = f(r)$, $r >= 2$ be a second order linear recurrence with constant coefficients. For some boundary conditions a_0 and a_1, the solution of the recurrence is $1 - 2r + 3.2^r$.

Determine a_0, a_1, c_1, c_2 and f(r). (The solution is not unique).

Ans. $a_0 = 1 - 2(0) + 3.2^n(0) = 2$

$a_1 = 1 - 2(1) + 3.2^n(1) = 5$

$$a_2 = 1 - 2(2) + 3.2^n(2) = 9$$

$$c_1 = 4a_1 + c$$

Q. 4. (b) Solve the recurrence relation :

$$a_r + 3a_{r-1} + 2a_{r-2} = F(r)$$

Where $F(r) = \{1, r = 2$

0, otherwise.

With the boundary conditions $a_0 = a_1 = 0$.

Ans. $a_r + 3a_{r-1} + 2a_{r-2} = 0$

$$\left(s^2 + 3s + 2\right)a_r = 0$$

$$(s+1)(s+2) = 0$$

$$s = -1, -2$$

$\therefore$ $$a_r = -\left(c_1 + c_2 2^r\right)$$

Put $r = 2$

$$a_2 = -\left(c_1 + c_2 2^2\right) = 1$$

$$a_2 = -(c_1 + 4c_2) = 1 \qquad ...(1)$$

Put r =0

$$a_0 = -(c_1 + c_2) = 0 \qquad ...(ii)$$

From (i) & (ii)

$$-(c_1 + 4c_2) = 1$$

$$-(c_1 + c_2) = 0$$

$$3c_2 = 1$$

$$c_2 = \frac{1}{3}$$

Put $c_2 = \frac{1}{3}$ in (i)

$$-c_1 - \frac{4}{3} = 1$$

$$-c_1 = 1 + \frac{4}{3}$$

$$-c_1 = \frac{7}{3} \Rightarrow c_1 = -\frac{7}{3}$$

$$\therefore \quad a_r = \left(-\frac{7}{3} + \frac{1}{3}.2^r\right)$$

$$\therefore \quad a_r = \frac{7}{3} - \frac{1}{3}.2^r \Rightarrow 3a_r = 7 - 1.2^r$$

Q. 5. (a) What do you mean by Eulerian Circuit and a Hamiltonian circuit?

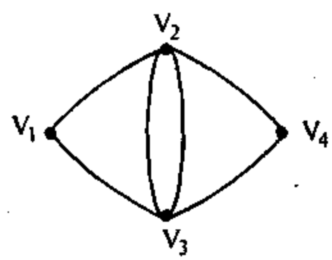
Ans. Hamiltonian Circuit : A Hamiltonian circuit is a path in which the initial vertex appears a second time as the terminal vertex.

Euler Circuit : Consider any connected planar graph G = (V, E) having R region-, V vertices and E edges,

$$V + R - E = 2$$

Q. 5. (b) Show a graph that has both an Eulerian Circuit and Hamiltonian circuit.

Ans.

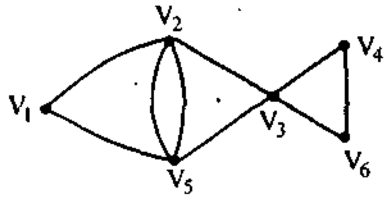


The euler circuit is $V_1, V_5, V_2, V_5, V_4 V_2, V_1$

The Hamiltonian circuit is V_1, V_2, V_4, V_5, V_1

Q. 5. (c) Show a graph that has an Eulerian circuit but has no Hamiltonian circuit.

Ans.

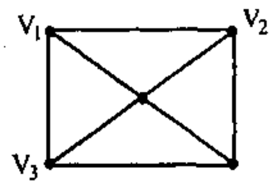


"Euler circuit is $V_1, V_5, V_2, V_5, V_4, V_6, V_3, V_2, V_1$

There is Hamiltonian circuit..

Q. 5. (d) Show a graph that has no Eulerian circuit but has a Hamiltonian circuit.

Ans.



Hamiltonian circuit is V_1, V_2, V_4, V_3, V_1

No Euler circuit.

Q. 6. (a) Draw all possible non similar trees T where :

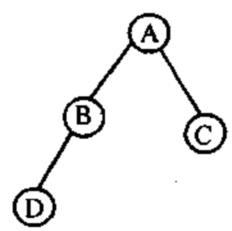
(i) T is a binary tree with 3 nodes.

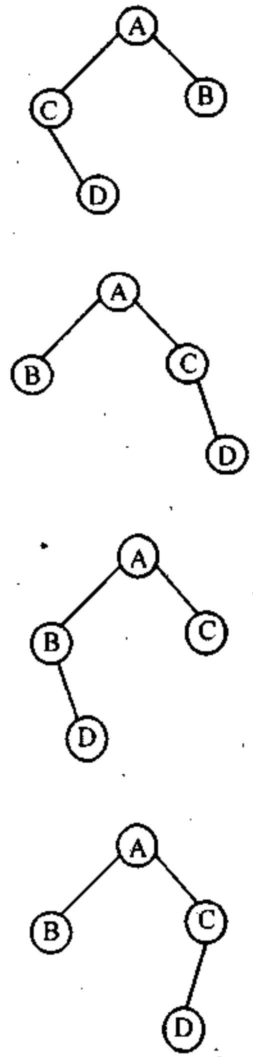
(ii) T is a 2-tree with 4 internal nodes.

Ans. (i) T is a binary tree with 3 node.

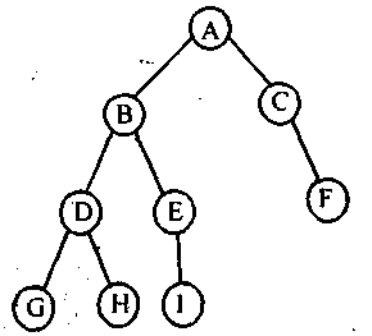
Suppose the tree T has one root and three childrens.

Say A is root and B, C, D are children of A.





(ii) A binary tree with 4 internal nodes.



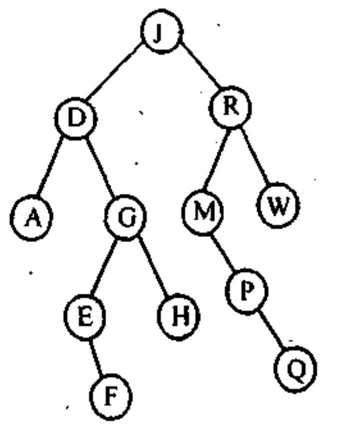
Q. 6. (b) Suppose the following list of literals is inserted in to any empty binary search tree

J, R, D, G, W, W, E, M, H, P, A, F, Q.

(i) Find the final tree T.

(ii) Find the in-order and pre-order traversal of tree T.

Ans. (i)



The above given tree is the final tree T from the given literals.

(ii) Inorder and preorder traversal in Inorder traversal we first traverse left node then root node then right node where as in pre order we traverse root-left-right node.

Inorder

ADEFGHDJMPQRW

Preorder

JDAGEFHRMPQW.

Q. 7. Write short notes on :

(a) Isomorphism and Automorphism

(b) Homomorphic and Isomorphic graphs

(c) Subgroups and Normal subgroups

(d) Equivalence relations and partitions.

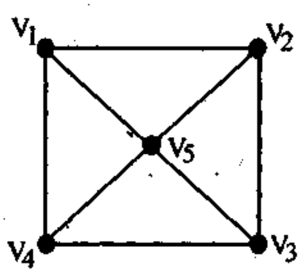
Ans. (a) Isomorphism : Let $(G_1,*)$ and $(G_2,*)$ be two algebraic system where * and 0 both are binary operations. The systems $(G_1,*)$ and $(G_2,0)$ are sand to be isomorphic if there exists an isomorphic mapping $f:G_1 \to G_2$. When two algebraic systems are isomorphic, the system are structbrally equivalent and one can be obtained from another by simply remaining the elements and the operation.

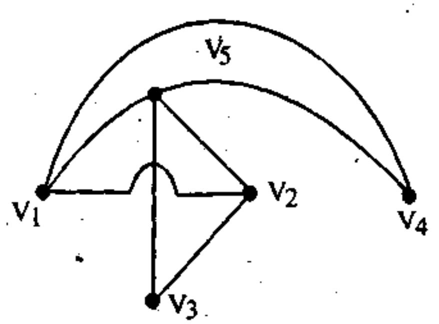
Automorphism : Let $(G_1,*)$ and $(G_2,*)$ be two algebraic systems, where * and 0 both are binary operations on G_1 and G_2 respectively. Then an isomorphism from $(G_1,*)$ and $(G_2,*)$ is called an automorphism if $G_1 = G_2$.

(b) Homomorphic : Let $(G_1,*)$ and $(G_2,*)$ be two algebraric systems, where * and 0 both are binary operations. Then the mapping $f:G_1 \to G_2$ is said to be homomorphism from $(a_1,*)$ to $(G_2,*)$ such that every a, $b \varepsilon G$ we have

$$f(a * b) = f(a).f(b)$$

Isomorphic Graphs : Two graph G_1 and G_2 are called isomorphic graphs if there is one to one correspondence between their vertices and between their edges.





(c) Subgroups : Let us consider a group (G, *). Also let $S \subseteq G$; then (S, *) is called a subgroup if it satisfies.

(i) The operation * is closed operation on S.

(ii) The operation * is an associative operation.

(iii) As e is identity element belonged to G. It must belong to the sets.

(iv) For every element $a \varepsilon S$, a^{-1} also belongs to S.

Normal Subgroup : consider a group (G, *) and subgroup (H, *) of the group, then the (H, *) is called a normal subgroup if any $a \varepsilon G$ when have,

$$aH = Ha$$

If H is normal subgroup, then both the left and right cosets of x G are equal.

(d) Equivalence Relations : A relation R on a set A is called an equivalence relation if it satisfies following three properties.

1. Relation R is reflexive i.e. $aRa \forall a \varepsilon A$.

2. Relation R is symmetric is $aRb \Rightarrow bRa$.

3. Relation R is transitive i.e. aRb and $bRc \Rightarrow aRc$.

Partition : A partition $(A_1, A_2, A_3, ..., A_\pi)$ of an non empty set A is defines as a collection of non-empty subsets of A such that,

(a) Every element of A belongs to one of A_i.

(b) If in and A_s are distinct, then $A_i \cap A_s = \phi$.

Q. 8. Let (A, *) be a semi group and e be a left identity. Furthermore, for every x in A three exists x ^ in A such that x^*x = e :

(a) Show that for any a, b, c in A if a *b = a*c then b = c.

(b) Show that (A, *) is a group by showing that e is an identity element.

Ans. (a) Taking the left hand side

$$a*b$$

Now left hand side will be equal to right hand side according to the cancellation property as $a, b, c \varepsilon A$ this property is valid.

So,

$a * b = a * c \Rightarrow b = c$ according to the left cancellation property.

Otherwise taking left hand side a*b = b*a (according to associative law).

Taking R:H:S

$$a*c = c*a$$ (again according to associative law)

Now $b*a = c*a \Rightarrow b = c$ according to right cancellation.

(b) Given that

$$x\wedge x = e$$

e is the identify iff.

$$e*(x\wedge *x) = e \qquad ...(1) \text{ (according to left identity)}$$

According to right identity

$$(x\wedge *x)*e = e \qquad ...(2)$$

From 1 & 2

$$e*(x\wedge *x) = (x\wedge *x)e$$

If fulfils the property of identity. So e is identity element of the group.