

## B. E.

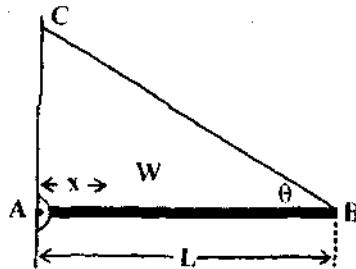
Third Semester Examination, May-2008

### Engineering Mechanics (ME-205, EA)

Note : Attempt any *FIVE* questions.

Q. 1. (a) A bar AB of negligible weight is pinned at A and supported at B by a thin wire BC. A weight W can move along the bar. Find :

- Tension in the wire,
- Horizontal reaction force and
- Vertical reaction force on the bar by the pin at A?



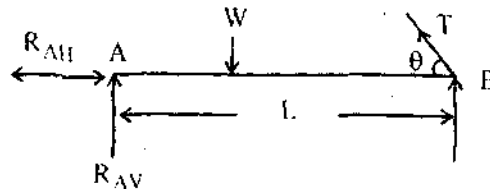
Ans. Let T is tension in wire BC.

Taking moment about A,

$$T \sin \theta \times L = W \cdot x$$

$\Rightarrow$

$$T = \frac{W \cdot x}{L \sin \theta}$$



$\therefore$

$$\Sigma F_v = 0$$

$$T \sin \theta - W + R_{AV} = 0$$

$$T \sin \theta - W + R_{AV} = 0$$

$\Rightarrow$

$$R_{AV} = W - T \sin \theta$$

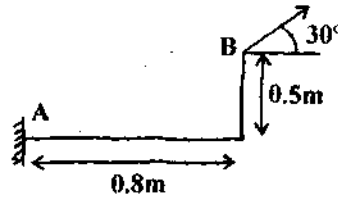
$$= W - \frac{Wx}{L} = W \left[ 1 - \frac{x}{L} \right]$$

$$\Sigma F_H = 0$$

$$R_{AH} = T \cos \theta$$

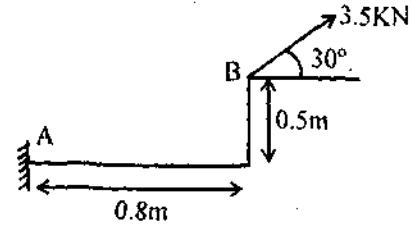
$$= \frac{Wx}{L} \cot \theta$$

Q. 1. (b) Find the moment of 3.5 kN force about A as shown in fig.

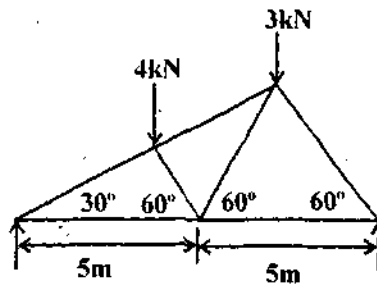


Ans. Moment about A,

$$\begin{aligned} &= 3.5 \cos 30^\circ \times 0.5 \\ &- 3.5 \sin 30^\circ \times 0.8 \\ &= 0.1155 \text{ kNm (clockwise)} \end{aligned}$$

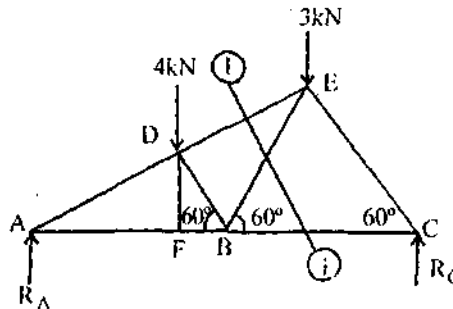


Q. 2. A truss of 10m span is loaded as shown in fig. Find the forces in the members of the truss by the method of sections.



Ans. Taking moment A,

$$R_C \times 10 = 3 \times 7.5 + 4 \times AF$$



Now,

$$AF = AB - FB = 5 - DB \cos 60^\circ = 5 - 5 \cos^2 60^\circ = 3.75 \text{ m}$$

$\Rightarrow$

$$R_C = 3.75 \text{ kN}$$

$\therefore$

$$R_A = 4 + 3 - 3.75 = 3.25 \text{ kN}$$

Taking section (1)-(1) as shown

$\therefore$

$$\Sigma M_E = 0$$

$$R_C \times 2.5 - F_{BC} \times 4.33 = 0$$

$$\Rightarrow F_{BC} = 2.16 \text{ kN (Tensile)}$$

$$\Sigma F_V = 0$$

$$R_C - 3 + F_{BE} \cos 30 - F_{DE} \cos 60 = 0 \quad \dots(i)$$

$$\Sigma F_H = 0$$

$$F_{BC} + F_{DE} \sin 60 - F_{BE} \sin 30 = 0 \quad \dots(ii)$$

Solving equations (i) & (ii) we get,

$$F_{BE} = 1.73 \text{ kN (Compressive)}$$

$$F_{DE} = -1.5 \text{ kN (Tensile)} = 1.5 \text{ kN (Compressive)}$$

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Taking joint C, we get  $F_{EC} = \frac{R_C}{\sin 60} = 4.33 \text{ kN (Compressive)}$

Taking joint A

$$F_{AD} \sin 30 = R_A$$

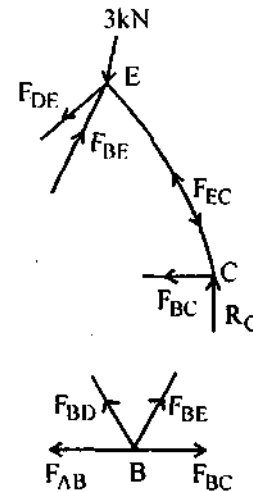
$$\Rightarrow F_{AD} = 6.5 \text{ kN (Compressive)}$$

$$F_{AB} = F_{AD} \cos 30 = 5.63 \text{ kN (Tensile)}$$

Taking joint B

$$F_{BD} \cos 30 = F_{BE} \cos 30$$

$$\Rightarrow F_{BD} = F_{BE} = 1.73 \text{ kN (Tensile)}$$



**Q. 3. (a) From a uniform circular disc of radius 'r' is cut off a circular portion of radius 'r/2'. Find the distance between the centers of the disc and hole if the center of the mass of the remainder is on the circumference of the hole.**

Ans. Let  $O_1$  &  $O_2$  are centres of circle of radius r and r/2 respectively.

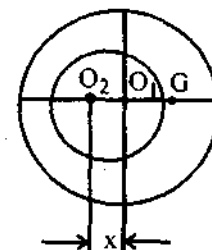
Let  $O_1O_2 = x$

'G' is the centre of max. such that

$$O_1G = \left( \frac{r}{2} - x \right) = \frac{A_1x_1 + A_2x_2}{A_1 - A_2}$$

$$\therefore \frac{r}{2} - x = \frac{\frac{\pi}{4} r^2 \cdot x}{\pi r^2 - \frac{\pi r^2}{4}}$$

$$\Rightarrow \frac{\pi}{4} r^2 x = \frac{\pi r^3}{2} - \pi r^2 x - \frac{\pi r^3}{8} - \frac{\pi r^3}{8} + \frac{2r^2}{4} x$$



$$\Rightarrow \pi r^2 x = \frac{3ar^3}{8}$$

$$\Rightarrow x = \frac{3r}{8}$$

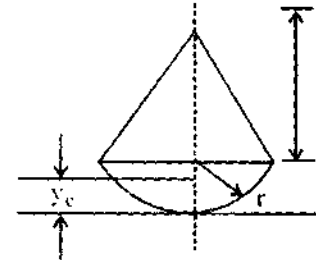
**Q. 3. (b)** A body consists of a right circular solid cone of height 20 cm and radius 15 cm placed on a solid hemisphere of radius 15 cm of the same material. Find the position of center of gravity.

**Ans.** 
$$y_c = \frac{V_1 y_1 + V_2 y_2}{V_1 + V_2}$$

$$V_1 = \frac{2}{3} \pi r^3, \quad V_2 = \frac{\pi}{3} hr^2$$

$$y_1 = r - \frac{3r}{8} = \frac{5r}{8}, \quad y_2 = r + \frac{h}{4}$$

$$y_c = \frac{\left(\frac{2\pi}{3} r^3\right)\left(\frac{5r}{8}\right) + \left(\frac{\pi}{3} r^2 h\right)\left(r + \frac{h}{4}\right)}{\frac{2\pi}{3} r^3 + \frac{\pi}{3} r^2 h}$$



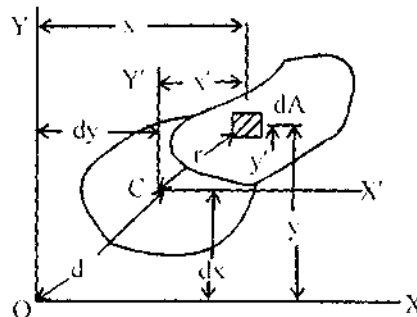
Putting  $r = 15$  cm and  $h = 20$  cm as given, we get

$$y_c = 13.625 \text{ cm.}$$

**Q. 4. (a)** State and prove the theorems of parallel axis and perpendicular axis.

**Ans. Parallel Axis Theorem :** It states that moment of Inertia of an area with respect to any axis in its plane is equal to moment of inertia of the area with respect to a parallel centroidal axis plus the product of area and square of distance between the two axes.

**Proof :** Let  $x, y$  be the rectangular coordinate axes through any point  $O$  in plane of figure.



**Q. 4. (b)** Derive an expression for moment of inertia of a quadrant of a circular plate of radius  $R$ .

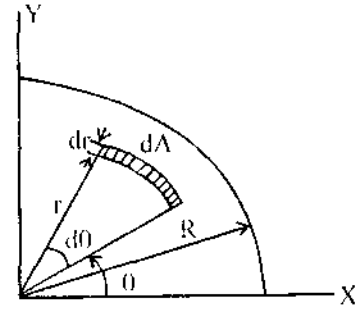
**Ans.** Consider elemental area  $dA$  situated at radius  $r$  and angle  $\theta$ .

$$dA = r d\theta dr$$

Centroid of  $dA$  lies at distance  $r \sin \theta$  from  $x$ -axis.

Moment of inertia about  $x$ -axis.

$$\begin{aligned}
 dI_x &= ((rd\theta)dr)(r\sin\theta)^2 \\
 I_x &= \int_{r=0}^{r=R} \int_{\theta=0}^{\theta=\pi/2} r^3 \sin^2\theta d\theta dr \\
 &= \int_0^R r^3 dr \int_0^{\pi/2} \left( \frac{1 - \cos 2\theta}{2} \right) d\theta \\
 &= \int_0^R r^3 dr \times \frac{1}{2} \left[ \theta + \frac{\sin 2\theta}{2} \right]_0^{\pi/2} \\
 &= \int_0^R \frac{\pi}{4} r^3 dr = \frac{\pi}{4} \cdot \frac{R^4}{4} = \frac{\pi R^4}{16}
 \end{aligned}$$



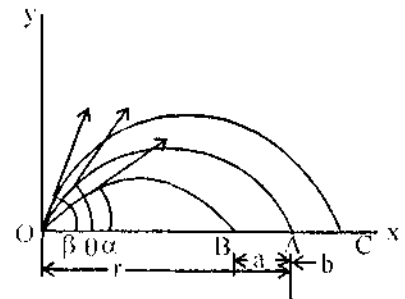
**Q. 5.** A projectile is aimed at a mark on the horizontal plane through the point of projection and falls 12m short when the angle of projection is  $15^\circ$ , while it overshoots the mark by 24 m when the same angle is  $45^\circ$ . Find the angle of projection to hit the mark. Assume no air resistance. Derive the expressions used.

**Ans.** Let  $v$  be the velocity of projection in each case and  $\theta$  be the correct angle of projection for desired range  $r$ . Let  $a = 12\text{m}$  when  $\alpha = 15^\circ$  and  $b = 24\text{m}$  when  $\beta = 45^\circ$ . Then

$$r - a = \frac{v^2 \sin 2\alpha}{g} \quad \dots(i)$$

$$r + b = \frac{v^2 \sin 2\beta}{g} \quad \dots(ii)$$

$$r = \frac{v^2 \sin 2\theta}{g} \quad \dots(iii)$$



of area  $A$  as shown.  $x', y'$  be corresponding parallel axes through centroid of area is also called centroidal axes. The moment of inertia of area  $A$  about  $x$ -axis,

$$I_x = \int y^2 dA$$

Where  $dA$  is elemental area at distance  $y$  from  $x$ -axis.

$$y = dx + y'$$

$$I_x = \int (y' + dx)^2 dA$$

$$= \int (y'^2 + dx^2 + 2y'dx) dA$$

$$= \int y'^2 dA + Adx^2 + 0$$

$$= I_x' + Adx^2 \quad \dots(i)$$

Similarly  $I_y = I_y' + Ady^2 \quad \dots(ii)$

**Perpendicular Axes Theorem :** It states that polar moment of Inertia i.e. moment of inertia about axis perpendicular to plane passing centroid of figure is equal to sum of moment of inertia about x and y axes parallel to plane passing centroid of figure.

**Proof :** Polar moment of inertia.  $J_0 = \int r^2 dA$

But  $r^2 = x^2 + y^2$

$$\therefore J_0 = \int (x^2 + y^2) dA = I_x' + I_y'$$

Subtract (i) from (ii)

$$a + b = \frac{v^2 \{ \sin^2 \beta - \sin^2 \alpha \}}{g} \quad \dots(iv)$$

Subtract (iii) from (ii)  $b = \frac{v^2 \{ \sin 2\beta - \sin 2\theta \}}{g} \quad \dots(v)$

From (iv) & (v)  $\frac{v^2}{g} = \frac{a + b}{\sin 2\beta - \sin 2\alpha}$

$$\frac{v^2}{g} = \frac{b}{\sin 2\beta - \sin 2\theta}$$

Equating,  $\frac{a + b}{\sin 2\beta - \sin 2\alpha} = \frac{b}{\sin 2\beta - \sin 2\theta}$

Solving

$$\therefore \theta = \frac{1}{2} \sin^{-1} \left[ \frac{a \sin 2\beta + b \sin 2\alpha}{a + b} \right]$$

Putting the values

$$\theta = 20.9^\circ$$

**Q. 6. (a) State and explain the D'Alembert's principle for the motion of rotation.**

**Ans.** D'Alembert's principle can be states as: "when internal or external turning moments act on a system having rotation motion, the algebraic jump of all torques acting on the system including the inertia torque, taken in opposite order of angular acceleration is zero."

Thus, as per D'Alemberts principle of rotation

$$T - I\alpha = 0$$

**Q. 6. (b) A solid cylinder pulley of mass 800 kg, having 0.8 m radius of gyration and 2m diameter, is rotated by an electric motor, which exerts a uniform torque of 60 kN.m. A body of mass 2000 kg is to be lifted by a wire wrapped round the pulley. Find :**

- (i) Acceleration of the body and  
 (ii) Tension in the rope.  
 Ans, Moment of Inertia of Pulley,

$$I_G = MK^2$$

$$= 800 \times (0.8)^2 = 512 \text{ kgm}^3$$

Also, acceleration

$$a = r\alpha = 1 \times \alpha = \alpha$$

Torque

$$T_1 = I_G \alpha = I_G \cdot a = 512a$$

Torque due to tension

$$T_2 = P \times r = P$$

Thus, total torque

$$= 512a + P = 60000$$

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$$P = 60000 - 512a$$

Equation of motion

$$P - mg = ma$$

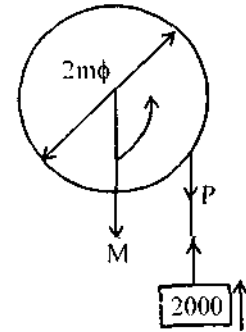
$$\Rightarrow 60000 - 512a - 2000 \times 9.8 = 2000 \times a$$

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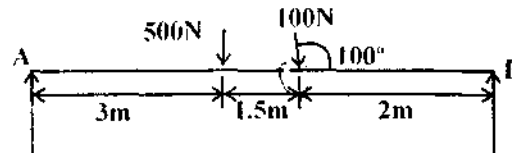
$$a = 16.08 \text{ m/s}^2$$

Tension is given by,

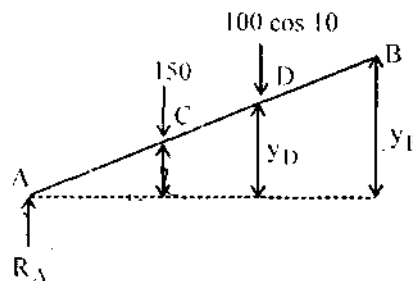
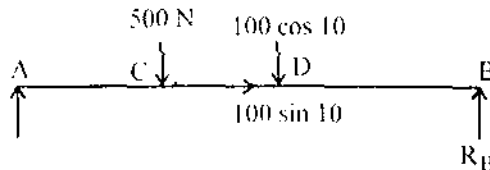
$$P = 2000 \times (9.8 + 16.08) = 50 \text{ KN}$$



Q. 7. Explain the principle of Virtual Work. Solve the beam for reactions at supports show in fig., by virtual work method.



Ans. Principle of virtual work states that if a system of forces acting on a body or a system of bodies be in equilibrium and if the system be imagined to undergo a small displacement consistent with geometrical conditions then algebraic sum of virtual work done by the forces of system is zero.



$$\frac{y_B}{y_C} = \frac{AB}{AC} = \frac{6.5}{3}$$

$$\Rightarrow y_B = 2.16 y_C$$

$$\frac{y_D}{y_C} = \frac{AD}{AC} = \frac{4.5}{3}$$

$$\Rightarrow y_D = 1.5 y_C$$

As all virtual work principle.

$$-500 \times y_C + 100 \cos 10^\circ \times y_D + R_B \times y_B = 0$$

$$R_B = \frac{500 + 100 \cos 10^\circ \times 1.5}{2.16}$$

$$= 299.8 \text{ N}$$

$$R_A = 500 + 100 \cos 10^\circ - 299.8$$

$$= 298.6 \text{ N}$$

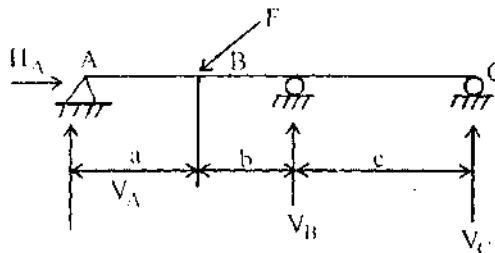
**Q. 8. Discuss the following with suitable examples :**

(a) Static indeterminacy,

(b) Hamilton principle,

(c) Coriolis forces.

**Ans. (a) Static Indeterminacy :** A system can be statically indeterminate even though its reactions are determinate. A structure is statically indeterminate when the static equilibrium equations are not sufficient for determining the internal forces and reactions on that structure.



For example, the equilibrium equations for the force system as shown are.

$$V_A - F_V + V_B + V_C = 0$$

$$H_A - F_h = 0$$

$$F_V \cdot a - V_B(a + b) - V_C(a + b + c) = 0$$

There are four unknowns and three equations. Thus, system is said to static indeterminate.

**(b) Hamilton Principle :** It states that the true evolution  $q(t)$  of a system described by  $N$  generalised coordinates  $q = (q_1, q_2, q_3, \dots, q_n)$  between two specified states  $q_1 = q(t_1)$  and  $q_2 = q(t_2)$  at two specified times  $t_1$  and  $t_2$  is an extremum of the action functional,



$$S[q(t)] = \int_{t_1}^{t_2} L(q, \dot{q}, t) dt$$

**(c) Coriolis Forces :** The Coriolis effect is an apparent deflection of moving objects when they are viewed from rotating frame of reference. The Coriolis force is an example of fictitious force because it does not appear when the motion is expressed in an inertial frame of reference, in which the motion of an object is explained by real impressed forces together with inertia. In rotating frame the Coriolis force which depends on the velocity of moving object and centrifugal force which doesn't depend on the velocity of moving object are needed in the equation to correctly describe the motion. The mathematical equation for Coriolis force is,

$$F_c = -2m\omega \times v$$

Where  $\omega$  = angular velocity  
and  $v$  = velocity of particle.